Study of stellar black holes with a gravity-like attractive potential in Bose-Einstein condensates

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Abstract

The paper deals with the problem that Bose-Einstein Condensate (BEC) with positive scattering length of S-wave of alkali metal atom simulates a one-dimension isolated static stellar black hole at primary level, under 1/R type of attractive potential in manners of "laser beam bathing" by radiation of six groups of laser including eighteen beams of laser. The results of our theory model have been proved feasible. We introduce the coefficient of loss ratio of three body recombination collision and the variational tendency of critical density and show entropy are discovered.

Keywords: BEC; Laser; Black Hole; Density; Entropy

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I. INTRODUCTION

By so far, Bose-Einstein Condensate (BEC) has been observed in the experiments [1–3]. It has been possible to manipulate neutral atom, with the breakthrough of the experiment. A great number of theory study have followed emerging. Most of these studies focus on the interaction characterized in S-wave scattering length between atoms. However, the study of the attractive interaction between atoms, with positive S-wave scattering length, is very poor. We think that some related physical problems are worth studying, such as some astrophysical problems including black hole governed by gravity, under the illumination of the study [4] of 1/R typical attractive interaction stimulated by laser radiation in BEC by D.O., 'Dell recently. James Angelin have appraised and affirmed the outstanding work of D.O., 'Dell in his paper [5]. Otherwise, A set of excellent studying fruits, such as, Relativistic effects of light in moving media with extremely low group velocity by U.Leonhurdt and P.Piwnicki [6], Spinor condensate observation in experimental level by J.Stenger [7], Two componential circuit Vortex’s observation in experimental level by M.R.Mutthews [8], and the technique of four wave mixing with frequency by Deng, L [9], convince us that the simulation of the phenomena governed by gravity at the experimental level, such as stellar black hole is a feasible fact. So, The paper focuses on the study of some basic phenomena of the simulation of one-dimension isolated, static, non-circumrotational black hole.

II. OUTLINE OF PHYSICAL THOUGHTS

A 1/R type of potential of attractive interaction [4]

we assume $c = \hbar = k_B = 1$ in the following.

The laser wavelengths must be longer than the size of the sample. The laser frequencies must be far from any resonant frequencies of the atoms. The laser fields must be intense laser fields. Bathing a ultra-cold atom cluster (BEC) in eighteen beams of round polarization
laser of three pairs with six groups. Every laser beam group includes three beams of laser, and every two laser beams in one group keep perpendicular polarization direction. The spatial distribution of the laser beam assembly presents the following Euler angle distribution. They are: 

\((0, \frac{\pi}{4}, \frac{\pi}{8}), (0, \frac{\pi}{4}, -\frac{\pi}{8}), (0, \frac{3\pi}{8}), (0, 0, \frac{\pi}{8}), (0, 0, -\frac{\pi}{8})\). The last two triads should be of half the intensity \(I\) of the others;

\[\delta \ll KL, \Delta I/I \ll KL.\]

\(\delta\) is the error of angular misalignment between interperpendicular lasers. \(L\) is average radius of the condensate.

\[\rho \times a^3 \ll 1, \rho \times a^{3*} \gg 1, a^* \geq \lambda_{db} \geq a.\]

\(a^* = \frac{1}{4\pi^2 m_u}\) is the Bohr radius associated with the gravitational-like coupling. The means overcoming the interferences among all relative laser beam triads should be: first, introducing frequency shifts between the laser beams. Spreading the frequencies \(\omega_n\) of the \(E_n\) laser fields in intervals about the central frequency. Second, \(|\omega_n - \omega'_n| \geq 10^4 Hz\). Third, making the \(r^{-3}\) interference terms in the interaction energy \(\propto E_N E_{n'} (n \neq n')\) oscillate at the difference frequencies \(|\omega_n - \omega'_n|\).

**B Contrast of physical image between BEC under inducing attractive interaction and stellar black hole**

Formulae of attractive interaction BEC: \(F_G = -\nabla \left[ Nu \int \frac{|\psi(r')|^2}{|r-r'|} dr' \right];\)

Formulae of Stellar black hole: \(F_G = -G \frac{m_1 m_2}{r^2}.\)

The simulated strength of gravity-like \(u\) is \(10^{17}\) times more than that of the gravity. It seems reasonable for us to simulate the evolutive of the Universe because the evolutive speed of Universe is proportion to strength of gravity. In fact, BEC is a dynamic evolutive process. The representative BEC has undergone three stages that are the critical states before BEC, After BEC, and Almost pure BEC after evaporation cooling. The third stage is the focus of our interests. The speciality of the stage is the population of atoms of
condensate occupies an absolute larger fraction than that of atoms of heat background gas. The typical BEC can last 15 seconds, with the average spatial size of 10\(\mu m\). After having shut off Magneto-optical trap (MOT), opening the sources of laser beam group, the 1/R type of attractive interaction potential engenders, under suitable spatial intensity, and frequency arrangement of laser beam group.

During the BEC contracts continuously, condensate state exchanges matter and energy with around surrounding. At some singular points, the density of atoms of BEC arrives at the so-called situation of saturation. The partial vacuum’s curvature becomes maximum. The effects of gravity lens-like might come into being. The inelastic collision’s section enlarges when BEC contracts continuously. The critical numerical value of entropy presents increasing trend.

In describing methods, adopting Bogoliubov approximate method, and the technique of the shell truncation. The mechanism maintaining steady depends on the balance among degenerate atomic gas pressure, gravity-like and, other interactions.

One of important results of Einstein’s principle of general relativity are the existence of gravity collapsing. That is, stellar-black hole, and the radiation of gravity from matter dynamic distribution. In fact, The real stellar black hole represents a kind of dynamical evolitional process, following matter and energy’s exchange with surrounding[10–12]. In general, The critical size of black hole is limited in 2GM. When the characteristic length is below 2GM, Einstein’s principle of relativity lapses in dealing with the problems of black hole. The quanta effects begin occupying dominant position. The density of compact matter is enlarged continuously when the inflation of quality of black hole come into being. The high degenerate pressure of particle current fails to support their quanta gravity. So, The phenomenon of evaporation engenders, and furthermore, leading the phenomenon of diffusing extending. In describing methods, Some quantum mechanics models and local density
approximate means have wide application in the field[13–15].

III. MODEL OF THEORY

A The Hamiltonian and Equation of Density

The simplified Hamiltonian of the static isolated black hole-like in spherical symmetrical space

$$H = -\frac{1}{2m} \times \Delta + Nu \int \frac{\psi^2(r')}{|r - r'|} dr', \quad (1)$$

where $m$ is quality per atom of alkali metals, $N$ is total ultra-cold atomic number, $u = -\frac{112K^2I}{\varepsilon_0}$ is Intensity of induced gravity, $\psi$ is wave function, $\alpha$ is coupled constant, $K$ is wave sector, and $I$ is intensity of laser. Introducing Schwartzchild metric, we consider only near one-dimension light prick’s situation. When

$$ds^2 \geq 0, \quad (2)$$

we can gain the following formulae:

$$\frac{dr}{dt} = \left(1 - \frac{2uN}{mr}\right)^2. \quad (3)$$

Now we introduce Klein-Gordon equation:

$$E^2\psi(r, t) = H^2\psi(r, t). \quad (4)$$

We only consider one dimension static radial situation. The two sides of equation(4) multiplied by $\psi^*(r)$ from the right side. We can acquire the following equation of density.

$$H^2\varphi(r) = E^2\varphi(r), \quad (5)$$

where $\varphi(r) = |\psi(r)|^2$ Combining equation(1-5), We can have the following Klein-Gordon-like equation of density

$$-\left(1 - \frac{2uN}{mr}\right)^2 \frac{d^2\varphi(r)}{dr^2} = \left[-\frac{1}{2m} \frac{d^2}{dr^2} + Nu \int \frac{\varphi(r')}{|r - r'|} dr'\right]^2 \varphi(r). \quad (6)$$
One dimension spherical symmetrical static stellar black hole-like involved in the effect of three body recombination collision

$$H = -\frac{1}{2m} \times \Delta - Nu \int \frac{\psi(r')^2}{|r - r'|} dr' + \frac{4\pi a}{m} \times |\psi(r)|^2 - \mu l \int |\psi(r')|^6 dr'. \quad (7)$$

The parameters such as $H, m, N, u, a, \psi, \alpha, K, I$ have the same meaning as these of $1 >$. And $\mu$ is Chemical potential, $l$ is Coefficient of loss ratio of three body recombination collision. The technique of setting up equation of density is similar to that of $1 >$. It’s density equation has the following manners:

$$- \left(1 - \frac{2uN}{mr}\right)^2 \frac{d^2 \varphi(r)}{dr^2} = \left[-\frac{1}{2m} \frac{d^2}{dr^2} + Nu \int \frac{\varphi(r')}{|r - r'|} dr' + \frac{4\pi a}{m} \times \varphi(r) - \mu l \int \varphi(r')^3 dr'\right]^2 \varphi(r). \quad (8)$$

**B Results of solution**

Density and entropy functions of the first situation.

We adopt $\varphi(r) \sim e^{-r^2}$ as trial function of density according to the character of BEC.

When $r$ is not equal to 0, or 0.3525, We can acquire the following results

$$\varphi(r) \sim (\pi umN)^{-2} \frac{e^{-r^2}}{(r^2 - 0.5)^2} \left[-16r^{10} + (24 - 16m^2)r^8 + (24 + 78.176uNm)r^7ight.

- (116 + 64N^2u^2 - 200m^2)r^5 + (138 - 42m^2 + 24u^2N^2)r^4 + (54 + 254.176umN)r^3

- (27 - 18m^2 + 48u^2N^2)r^2 - 50.7359r\right]. \quad (9)$$

When $r$ tends to zero, We can acquire $\varphi(r) = A \cos \frac{\pi mr}{12}$. When $r$ tends 0.3535, We have the following density function:

$$\varphi(r) = \frac{2 \sin (5.6569u - 2m)r}{(2m - 5.6569u)^2} + \varphi(0). \quad (10)$$

The bound condition can be acquired from Virial theorem and energy function, It is that the critical density function is $\varphi_c(r) = \left(\frac{45}{12}\right)^3 \frac{M_c T_c^3}{4\pi(\alpha N)^2}$. $M_c$ is critical quality, $T_c$ is the critical
temperature. When \( r \) is equal to zero in equation (9), we have

\[
\varphi(0) = \frac{4K}{(\pi umN)^2},
\]

\( k \) is integral constant. It is apparent that the formulae \( T \propto \frac{1}{M^2} \) is tenable. The situation is coincident with that of black hole. Otherwise, the expression of the critical entropy is:

\[
S_C \sim K(\pi umN)^{-2}(0.3333r^4 + r^2 - r - ln(r^2 - 0.5)^2),
\]

(11)

\( K \) is integral constant. The maximum entropy is:

\[
S_{max} \sim -4.417043067k_B(\pi umN)^{-2}.
\]

(12)

The critical radius is \( R_c = 0.7825429(amu)^{-1} \)

Density and entropy Critical Numerical Value of the second situation are represented as following Table I and Table II

We choose \( \varphi(r) \sim e^{-r^2} \) as trial function of density according to the character of BEC. We are interested in the amplitude of density expression. We deal with the condensate of \(^{87}\text{Rb} \) and \(^{23}\text{Na} \), and contrast with the calculated results of them. Adopting the choice of regarding atomic quality unit(amu) as the basic unit. The unit of Strength of Stimulated gravity(\( u \))is \( amu^{-2} \).

The first group of critical numerical solution is expressed in the Table I.

<table>
<thead>
<tr>
<th>( \text{u (amu}^{-2} )</th>
<th>( \text{R}_C(amu^{-1}) )</th>
<th>( \varphi_C(amu^4) )</th>
<th>( S_C(amu^4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2.96800457 \times 10^{-9} )</td>
<td>( 6.644 \times 10^{-8} )</td>
<td>( 5.238 \times 10^{10} )</td>
<td>( -3.962 \times 10^{26} )</td>
</tr>
<tr>
<td>( ^{23}\text{Na} )</td>
<td>( 3 \times 10^{-10} )</td>
<td>( 6.716 \times 10^{-9} )</td>
<td>( 5.238 \times 10^{10} )</td>
</tr>
<tr>
<td>( 3 \times 10^{-8} )</td>
<td>( 6.716 \times 10^{-7} )</td>
<td>( 5.238 \times 10^{10} )</td>
<td>( -3.962 \times 10^{26} )</td>
</tr>
</tbody>
</table>

| \( 2.96800457 \times 10^{-9} \) | \( 9.506 \times 10^{-7} \) | \( 1.982 \times 10^{11} \) | \( -7.287 \times 10^{77} \) |
| \( ^{87}\text{Rb} \) | \( 3 \times 10^{-10} \) | \( 9.609 \times 10^{-8} \) | \( 1.982 \times 10^{11} \) | \( -7.287 \times 10^{77} \) |
| \( 3 \times 10^{-8} \) | \( 9.609 \times 10^{-6} \) | \( 1.982 \times 10^{11} \) | \( -7.287 \times 10^{77} \) |

Noting: \( n = 10000, a = 0.2880536amu^{-1}, l = 2.0374 \times 10^{30}amu^{-5}, \mu = 9.759481 \times 10^{-21}amu \)
In addition to the above result, there is an extreme solution in the first situation. Assuming $u = 0.296800457 \times 10^{-8}$ amu$^{-2}$, we get the solution:

$$^{23}\text{Na} R_{ec} = 0.7071068 \text{amu}^{-1} \quad ^{87}\text{Rb} R_{ec} \sim 0.7071068 \text{amu}^{-1}$$

$$^{23}\text{Na} \varphi_{ec} \sim 1.8818855 \times 10^{11} \text{amu}^4 \quad ^{87}\text{Rb} \varphi_{ec} \sim 1.8818855 \times 10^{11} \text{amu}^4$$

$$^{23}\text{Na} S_{ec} \sim 5.20337762 \times 10^{10} \text{amu}^4 \quad ^{87}\text{Rb} S_{ec} \sim 5.20337762 \times 10^{10} \text{amu}^4$$

The second group of critical numerical solution is expressed in the Table II.

<table>
<thead>
<tr>
<th></th>
<th>$u$ (amu$^{-2}$)</th>
<th>$R_C$ (amu$^{-1}$)</th>
<th>$\varphi_C$ (amu$^4$)</th>
<th>$S_C$ (amu$^4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{23}\text{Na}$</td>
<td>$2.96800457 \times 10^{-9}$</td>
<td>$1.267 \times 10^{-8}$</td>
<td>$1.000$</td>
<td>$0$</td>
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<tr>
<td></td>
<td>$3 \times 10^{-10}$</td>
<td>$1.281 \times 10^{-7}$</td>
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</tr>
<tr>
<td></td>
<td>$3 \times 10^{-8}$</td>
<td>$1.281 \times 10^{-9}$</td>
<td>$1.000$</td>
<td>$0$</td>
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<tr>
<td>$^{87}\text{Rb}$</td>
<td>$2.96800457 \times 10^{-9}$</td>
<td>$8.448 \times 10^{-7}$</td>
<td>$1.000$</td>
<td>$0$</td>
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<tr>
<td></td>
<td>$3 \times 10^{-10}$</td>
<td>$1.281 \times 10^{-7}$</td>
<td>$1.000$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>$3 \times 10^{-8}$</td>
<td>$1.281 \times 10^{-9}$</td>
<td>$1.000$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Noting: The unit of Strength of Stimulated gravity ($u$) is amu$^{-2}$, $n = 10000$, $a = 0.2880536$ amu$^{-1}$, $l = 2.0374 \times 10^{30}$ amu$^{-5}$, $\mu = 9.759481 \times 10^{-21}$ amu.

### IV. DISCUSSION AND CONCLUSION

After contrasting solution one, solution two, and extreme situation about $^{23}\text{Na}$ and $^{87}\text{Rb}$ condensate under the stimulated gravity-like, We can deduce the following deduction:

Both solution one and solution two present the tendency that the critical value of density decreases while the critical radius increases. In the meantime, The critical value of entropy also decreases when the critical radius increases. But, The decreasing range exists great difference between the two situation. The results approve that the solution one and solution two locate at two different transition points. The result is similar to the practical situation of black hole. The first solution locates at the critical region of entropy, It can be analogy
as growth disc of black hole-like. The second solution locates at the outer horizon of the condensate. The extreme solution locates at the collapse region of black hole-like. The calculating results can be fit for the requirement of the experiment by adjusting intensity of laser, frequency of laser, and distribution of group of laser beam. Both the density function of solution one and solution two are periodic function. The plural density function represents the fact that our model reflects the basic situation of the black hole. That is, the exchange of matter and energy is originated from the plural density function.

The negative maximum entropy represents that forming growth disc induced by gravity-like is possible under the appropriate control of the intensity, frequency and spatial arrangement of laser group.

The model’s conclusion of quality being inverse ratio of temperature is coincident with the real stellar black hole.

The paper discusses about the basic situation of isolated and general static stellar black hole-like from experimental level. The results show that our theory model is feasible. Some calculating results exist some little deviation from these of the real stellar black hole, The problems show that our model needs to be improved furthermore, Adding some additional factors, such as spin, topological circuit. These problems will be overcome by us in our future work.

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